scalar product

[SQA] 1. The vectors a, b and c are defined as follows:

$$a = 2i - k, \quad b = i + 2j + k, \quad c = -j + k.$$

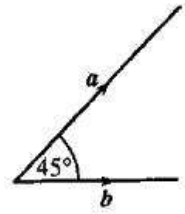
(*a*) Evaluate $a.b + a.c$. 3

(*b*) From your answer to part (*a*), make a deduction about the vector $b + c$. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CN	G18, G26		1993 P1 Q12
(*b*)	2	A/B	CN	G27		

- •1 $a = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, c = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$
- •2 $a.b = 1$
- •3 $a.c = -1$
- •4 $a.b + a.c = a.(b + c)$
- •5 $a \perp b + c$

[SQA] 2. The diagram shows two vectors a and b, with $|a| = 3$ and $|b| = 2\sqrt{2}$.
These vectors are inclined at an angle of 45° to each other.



(*a*) Evaluate (i) $a.a$
(ii) $b.b$
(iii) $a.b$ 2

(*b*) Another vector p is defined by $p = 2a + 3b$.
Evaluate $p.p$ and hence write down $|p|$. 4

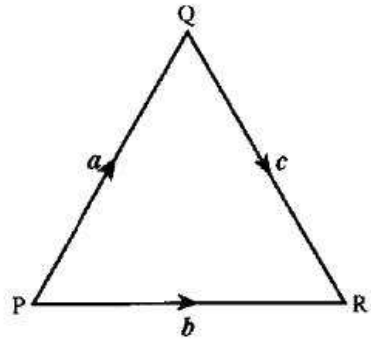
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G26		1999 P1 Q17
(*b*)	4	A/B	CN	G29, G30		

- •1 $a.a = 9$ and $b.b = 8$
- •2 $a.b = 6$
- •3 $(2a + 3b).(2a + 3b)$
- •4 $4a.a + 9b.b + 12a.b$
- •5 180
- •6 $\sqrt{180}$

[SQA] 3. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = a$, $\overrightarrow{PR} = b$ and $\overrightarrow{QR} = c$.

Evaluate $a.(b+c)$ and hence identify two vectors which are perpendicular. 4



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	CN	G26		1997 P1 Q13
	3	A/B	CN	G29, G27		

- •1 $a.b + a.c$
- •2 $a.b = 2 \times 2 \times \frac{1}{2}$
- •3 $a.c = 2 \times 2 \times -\frac{1}{2}$
- •4 0 and a is perpendicular to $(b+c)$

[SQA] 4. Vectors p, q and r are defined by

$p = i + j - k$, $q = i + 4k$ and $r = 4i - 3j$.

(a) Express $p - q + 2r$ in component form. 2

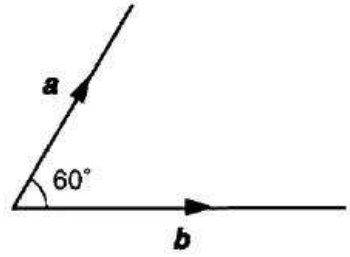
(b) Calculate $p.r$ 1

(c) Find $|r|$. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G16		1998 P1 Q3
(b)	1	C	CN	G26		
(c)	1	C	CN	G16		

- •1 $p = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $q = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$, $r = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$ s/i by •2
- •2 $\begin{pmatrix} 8 \\ -5 \\ -5 \end{pmatrix}$
- •3 1
- •4 5

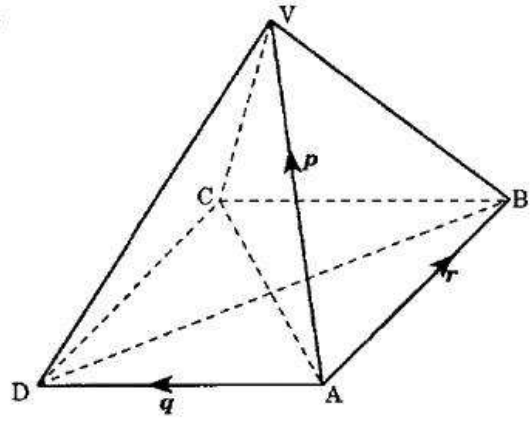
[SQA] 5. The diagram shows representatives of two vectors, a and b, inclined at an angle of 60°.
If $|a| = 2$ and $|b| = 3$, evaluate $a.(a + b)$ 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G29, G26		1992 P1 Q18

- •1 $a.a + a.b$
- •2 $2 \times 3 \times \cos 60°$
- •3 4

[SQA] 6. In the square-based pyramid, all the eight edges are of length 3 units.
$\vec{AV} = p$, $\vec{AD} = q$, $\vec{AB} = r$.
Evaluate $p.(q + r)$. 4

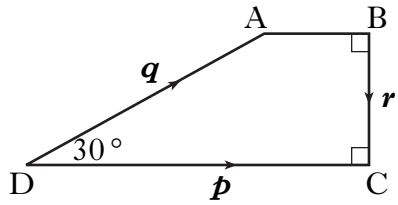


Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	CN	G26		1995 P1 Q16
	3	A/B	CN	G29, G26		

- •1 $p.q + p.r$
- •2 $V\hat{A}D = 60°$ or equiv.
- •3 $|p||q|\cos V\hat{A}D + |p||r|\cos V\hat{A}B$
- •4 9

- •1 $r = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, q = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$
- •2 $p = \begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \\ \frac{3}{\sqrt{2}} \end{pmatrix}$
- •3 $\left(-\frac{3}{2}\right) \times (-3) + \left(\frac{3}{2}\right) \times 3 + \frac{3}{\sqrt{2}} \times 0$
- •4 9

[SQA] 7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC $= 30°$.



It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.

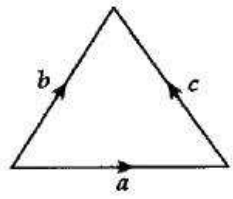
(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})$. **6**

(*b*) Find $|\boldsymbol{q}+\boldsymbol{r}|$ and $|\boldsymbol{p}-\boldsymbol{q}|$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	6	B	CN	G29, G26	$6\sqrt{3}$, $\frac{9}{4}$	2009 P2 Q7
(*b*)	2	A	CR	G21, G30	$\lvert\boldsymbol{q}+\boldsymbol{r}\rvert = \frac{3\sqrt{3}}{2}$	
(*b*)	2	B	CR	G21, G30	$\lvert\boldsymbol{p}-\boldsymbol{q}\rvert = \sqrt{(4-\frac{3\sqrt{3}}{2})^2+(\frac{3}{2})^2}$	

•[1] ss: use distributive law	•[1] $\boldsymbol{p}.\boldsymbol{q}+\boldsymbol{p}.\boldsymbol{r}$
•[2] ic: interpret scalar product	•[2] $4\times 3\cos 30°$
•[3] pd: processing scalar product	•[3] $6\sqrt{3}(\approx 10{\cdot}4)$
•[4] ic: interpret perpendicularity	•[4] $\boldsymbol{p}.\boldsymbol{r}=0$
•[5] ic: interpret scalar product	•[5] $-\lvert\boldsymbol{r}\rvert\times 3\cos 120°$
•[6] pd: complete processing	•[6] $r=\frac{3}{2}$ and $\frac{9}{4}$
•[7] ic: interpret vectors on a 2-D diagram	•[7] $\boldsymbol{q}+\boldsymbol{r}\equiv$ from D to the proj. of A onto DC
•[8] pd: evaluate magnitude of vector sum	•[8] $\lvert\boldsymbol{q}+\boldsymbol{r}\rvert=\frac{3\sqrt{3}}{2}$
•[9] ic: interpret vectors on a 2-D diagram	•[9] $\boldsymbol{p}-\boldsymbol{q}=\overrightarrow{\text{AC}}$
•[10] pd: evaluate magnitude of vector difference	•[10] $\lvert\boldsymbol{p}-\boldsymbol{q}\rvert=\sqrt{(4-\frac{3\sqrt{3}}{2})^2+(\frac{3}{2})^2}(\approx 2{\cdot}05)$

[SQA] 8. The sides of this equilateral triangle are 2 units long and represent the vectors a, b and c as shown.
Evaluate $a.(a+b+c)$.



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Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	NC	A6		1989 P1 Q9
	4	A/B	NC	G29, G26		

- ^{1} $a.a + a.b + a.c$
- ^{2} $a.a = |a||a|\cos 0$
- ^{3} $a.b = |a||b|\cos 60$
- ^{4} $a.c = |a||c|\cos 120$
- ^{5} 4

[END OF QUESTIONS]